

2022 年普通高等学校招生全国统一考试

数学

第 27 题

1. 2021 年普通高等学校招生全国统一考试数学第 27 题

已知函数 $f(x) = 3^x - 1$, $g(x) = x^2 - ae^x$. 若 $f(x) > g(x)$ 在 $x \in (0, 2)$ 上恒成立, 求 a 的取值范围.

A. $(-\infty, \frac{1}{e}]$ B. $(-\infty, \frac{4}{e}]$ C. $(-\infty, \frac{4}{e^2}]$ D. $(-\infty, \frac{2}{e^2}]$

解: 由 $f(x) = 3^x - 1 = 0$ 得 $x = 2$.

由 $g(x) = x^2 - ae^x = 0$ 得 $x^2 = ae^x$. 记 x_0 为方程 $x^2 = ae^x$ 的根.

由 $f(x) = 3^x - 1$, $g(x) = x^2 - ae^x$ 知 $f(x) > g(x)$ 在 $x \in (0, 2)$ 上恒成立.

$\therefore |x_0 - 2| < 1$, 即 $1 < x_0 < 3$.

由 $x_0^2 = ae^{x_0}$ 得 $a = \frac{x_0^2}{e^{x_0}}$.

记 $h(x) = \frac{x^2}{e^x}$, 则 $h(x) = \frac{2x - x^2}{e^x}$, $x \in (1, 3)$.

由 $1 < x < 2$ 得 $h(x) > 0$, $h(x)$ 在 $(1, 2)$ 上单调递增.

由 $2 < x < 3$ 得 $h(x) < 0$, $h(x)$ 在 $(2, 3)$ 上单调递减.

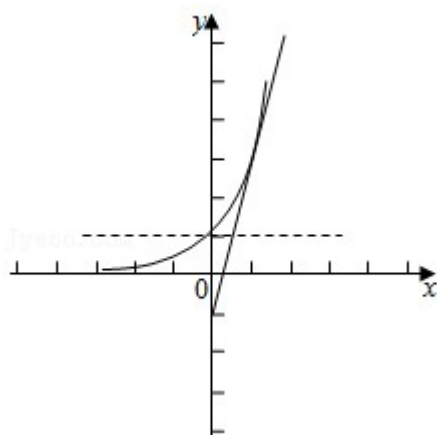
$\therefore h(x)_{\max} = h(2) = \frac{4}{e^2}$, $h(1) = \frac{1}{e}$, $h(3) = \frac{9}{e^3}$.

$\therefore a$ 的取值范围是 $(-\infty, \frac{4}{e^2}]$.

故选 B.

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2. 2021 年普通高等学校招生全国统一考试数学第 27 题



$2a = e^e$ $a = \frac{e^e}{2}$ $y = 2a(x-1)$ $y = e^x$

$2a > e^e$ $a > \frac{e^e}{2}$ $y = e^x$ $y = 2a(x-1)$

a $(\frac{e^e}{2}, +\infty)$

D

4. 2021 • $f(x) = (ax^3 - x^2 + 4a)\ln(\sqrt{x^2 + 1} + x)$ a ()

A $(0, 1)$

B $(0, \frac{1}{3})$

C $(1, +\infty)$

D $(\frac{1}{3}, +\infty)$

$g(x) = \ln(\sqrt{1+x^2} + x)$ $g(x)$ R

$g(-x) + g(x) = \ln(-x + \sqrt{x^2 + 1}) + \ln(x + \sqrt{x^2 + 1}) = \ln(x^2 + 1 - x^2) = \ln 1 = 0$

$g(x)$

$x = 0$ $y = x + \sqrt{x^2 + 1}$ $g(x)$ R $x > 0$ $g(x) > 0$ $x < 0$ $g(x) < 0$

$h(x) = ax^3 - x^2 + 4a$ $a = 0$ $h(x) = -x^2, 0$

$f(x)$

$a < 0$ $x > 0$ $h(x) < 0$ $g(x) > 0$ $f(x) < 0$ $x > 0$ $f(x)$

$a > 0$ $h(x) = ax^2 - x^2 + 4a$ $h(x) = 3ax^2 - 2x$

$h(x)$ 0 $\frac{2}{3a}$ $h(x)$ $(0, \frac{2}{3a})$ $(-\infty, 0)$ $(\frac{2}{3a}, +\infty)$

$h(x)$ $x=0$ $4a$ $x=\frac{2}{3a}$ $4a - \frac{4}{27a^2}$

$f(x)$

$4a > 0$ $4a - \frac{4}{27a^2} < 0$ $0 < a < \frac{1}{3}$

B

5 2021 $y = \frac{b}{|x| - c}$ ($c > 0, b > 0$)

$f(x) = a^{x^2 + x + 1}$ ($a > 0, a \neq 1$) $c=1$ $b=1$ $y = \log_a |x|$

A 1

B 2

C 4

D 6

$y = \frac{b}{|x| - c}$ ($c > 0, b > 0$)

$c=b=1$ $y = \frac{1}{|x| - 1}$

$f(x) = a^{x^2 + x + 1}$ ($a > 0, a \neq 1$)

$x^2 + x + 1 > 0$

$a > 1$

$y = \log_a |x|$

$c=1$ $b=1$ $y = \log_a |x|$ 4

C

$$f(x) = 2 \sin(\omega x + \varphi) \quad (\omega > 0, 0 < \varphi < \pi)$$

□□□□ $y=g(x)$ □□□□□□□□□□

① $\varphi = \frac{\pi}{3}$ \square

② $\square \square \mathcal{G}(X) \square \square \square \square \square \square \square \pi \square$

③ $\mathcal{G}(x)$ $[-\frac{\pi}{3}, \frac{\pi}{12}]$

④ $\mathcal{G}(X)$ $(-\frac{\pi}{3}, 0)$

□□□□□□□□□□ ()

A□4

B□3

C□2

D□1

$$\text{已知函数 } f(x) = 2\sin(\omega x + \varphi) (\omega > 0, 0 < \varphi < \pi) \text{ 的部分图象如图，求 } \omega \text{ 和 } \varphi \text{ 的值。}$$

$$T = \frac{2\pi}{\omega} > \frac{11\pi}{12}, \quad \frac{3}{4}T < \frac{11\pi}{12} \quad \therefore \omega \in \left(\frac{18}{11}, \frac{24}{11}\right)$$

$$f(0, \sqrt{3}) = 2\sin\varphi = \sqrt{3}$$

$$\therefore \varphi = \frac{\pi}{3} \quad \varphi = \frac{2\pi}{3}$$

$$\text{由 } \left(\frac{11\pi}{12}, 2\pi\right) \text{ 知 } \omega \cdot \frac{11\pi}{12} + \varphi = 2k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z}$$

$$\varphi = \frac{\pi}{3} \quad \omega \cdot \frac{11\pi}{12} + \frac{\pi}{3} = 2k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z} \quad \omega = \frac{2}{11} + \frac{24k}{11} \quad \omega \in \left(\frac{18}{11}, \frac{24}{11}\right)$$

$$\varphi = \frac{2\pi}{3} \quad A = 2$$

$$\omega \cdot \frac{11\pi}{12} + \frac{2\pi}{3} = 2k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z} \quad \omega = \frac{2}{11} + \frac{24k}{11}$$

$$k=1 \quad \omega=2 \quad \omega \in \left(\frac{18}{11}, \frac{24}{11}\right) \quad f(x) = 2\sin\left(2x + \frac{2\pi}{3}\right)$$

$$f(x) \text{ 的周期为 } \frac{\pi}{6}$$

$$y = g(x) = 2\sin\left(2x + \frac{\pi}{3}\right)$$

$$g(x) \text{ 的周期为 } \frac{2\pi}{2} = \pi \quad B = 2$$

$$x \in \left[-\frac{\pi}{3}, \frac{\pi}{12}\right] \quad 2x + \frac{\pi}{3} \in \left[-\frac{\pi}{3}, \frac{\pi}{2}\right] \quad g(x) \text{ 的周期为 } C = 2$$

$$x = -\frac{\pi}{3} \quad g(x) = -\sqrt{3} \neq 0 \quad g(x) \text{ 的周期为 } \left(-\frac{\pi}{3}, 0\right) \text{ 的周期为 } D = 2$$

$$2\pi$$

$$C = 2$$

$$y = A\sin(\omega x + \varphi) \text{ 的周期为 } \frac{2\pi}{\omega} \quad \varphi \text{ 的周期为 } \frac{2\pi}{\omega} \quad y = A\sin(\omega x + \varphi) \text{ 的周期为 } \frac{2\pi}{\omega}$$

$$2\pi$$

$$7 \times 2021 \cdot \text{已知函数 } M = \{1, 2, 3\} \quad N = \{1, 2, 3, 4\} \text{ 满足 } f: M \rightarrow N \text{ 满足 } A(1, f(1)) \in B(2, f(2))$$

已知 $\triangle ABC$ 中, D 为 BC 边上一点, $DA+DC=y$, $DB \in R$, 则 $f(x)$ 的值为 ()

- A 6 B 10 C 12 D 16

已知 $\triangle ABC$ 中, $BA=BC$, $f_1=f_3$, $f_1 \neq f_2$

则 $f_1=f_3=2$, $f_2=1$ 或 $f_1=f_3=3$, $f_2=2$

或 $f_1=f_3=4$, $f_2=1$

或 $f_1=f_3=1$, $f_2=2$

或 $f_1=f_3=2$, $f_2=3$

已知 $f(x)$ 的值为 12

已知 C

已知 $f(x)$ 的值为

8. 2021. 已知 $f(x)=2x-\cos x$, $\{a_n\}$ 满足 $\frac{\pi}{8} \leq a_1 < a_2 < \dots < a_5 \leq \pi$

则 $[f(a_5)]^2 - a_1 a_5 =$ ()

- A 0 B $\frac{1}{16}\pi^2$ C $\frac{1}{8}\pi^2$ D $\frac{13}{16}\pi^2$

已知 $f(x)=2x-\cos x$

则 $f(a_1)+f(a_2)+\dots+f(a_5)=2(a_1+a_2+\dots+a_5)-(\cos a_1+\cos a_2+\dots+\cos a_5)$

已知 $\{a_n\}$ 满足 $\frac{\pi}{8} \leq a_1 < a_2 < \dots < a_5 \leq \pi$

则 $a_1+a_2+\dots+a_5=5a_3$

$\cos a_1+\cos a_2+\dots+\cos a_5$

$=(\cos a_1+\cos a_5)+(\cos a_2+\cos a_4)+\cos a_3$

$=[\cos(a_3-\frac{\pi}{8})+\cos(a_3+\frac{\pi}{8})]+[\cos(a_3-\frac{\pi}{8})+\cos(a_3+\frac{\pi}{8})]+\cos a_3$



D-5

 $\square\square\square C \square$

$$OM \cdot ON = 40 \left(\frac{5}{a} + \frac{1}{b} \right)$$

A $\frac{25}{6}$

B $\frac{9}{4}$

C 1

D 4

由 $OM \cdot ON = ax + by$

$\therefore z = ax + by$ 中 z 的最小值为 40

故答案为：40

由 $z = ax + by$ 得 $y = -\frac{a}{b}x + \frac{z}{b}$

由 $y = -\frac{a}{b}x + \frac{z}{b}$ 得 $A(-\frac{a}{b}, \frac{z}{b})$

由 $z = ax + by$ 得 $z = 8a + 10b$

$\begin{cases} 2x - y - 6 = 0 \\ x - y + 2 = 0 \end{cases}$ 解得 $\begin{cases} x = 8 \\ y = 10 \end{cases}$

故 $A(8, 10)$

由 $z = ax + by$ 得 $40 = 8a + 10b$

$\frac{a}{5} + \frac{b}{4} = 1$

$\therefore \frac{5}{a} + \frac{1}{b} = \left(\frac{5}{a} + \frac{1}{b}\right)\left(\frac{a}{5} + \frac{b}{4}\right) = 1 + \frac{1}{4} + \frac{5b}{4a} + \frac{a}{5b} \geq \frac{5}{4} + 2\sqrt{\frac{5b}{4a} \cdot \frac{a}{5b}} = \frac{5}{4} + 2 \times \frac{1}{2} = \frac{9}{4}$

故 $\frac{5b}{4a} = \frac{a}{5b}$ 得 $4a^2 = 25b^2$ 得 $2a = 5b$

$\therefore \frac{5}{a} + \frac{1}{b} = \frac{9}{4}$

故 B

□ □ □ □ □ □ □

15□□202

A ☐

B□

C□

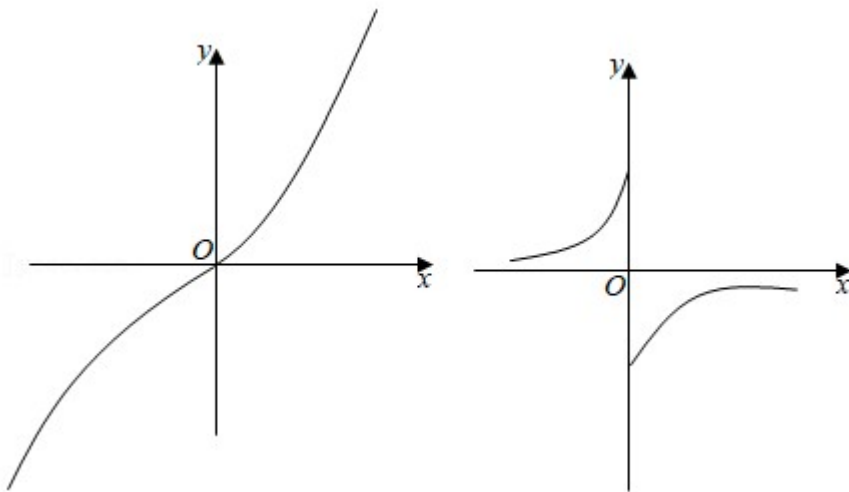
D□

□□□□□□

$$B: A(x)$$

$C: A \rightarrow B$

$D: A(x)$



函数在 R 上

有 C

函数在 R 上

16. 2021 年 1 月 1 日起, 我国将全面实施个人所得税综合所得汇算清缴, 其规则如下: 居民个人的综合所得, 以每一纳税年度的收入额减除费用 6 万元以及专项扣除、专项附加扣除和依法确定的其他扣除后的余额, 为应纳税所得额。已知某居民个人 2020 年应纳税所得额为 x 元, 且 $x \in [0, \pi]$, 则其应纳税额 y 元为 ()

A. $[\frac{41}{12}, \frac{15}{4})$

B. $(\frac{49}{12}, \frac{23}{4}]$

C. $(\frac{41}{12}, \frac{15}{4}]$

D. $[\frac{49}{12}, \frac{23}{4})$

函数 $f(x) = 2\cos^2(\omega x - \frac{\pi}{12}) - \frac{1}{2}$

$$= 2 \times \frac{\cos(2\omega x - \frac{\pi}{6}) + 1}{2} - \frac{1}{2}$$

$$= \cos(2\omega x - \frac{\pi}{6}) + \frac{1}{2}$$

若 $x \in [0, \pi]$

$$-\frac{\pi}{6} \leq 2\omega x - \frac{\pi}{6} \leq 2\omega\pi - \frac{\pi}{6}$$

$$y = \cos(2\omega x - \frac{\pi}{6}) \quad y = -\frac{1}{2}$$

函数 $f(x)$ 在 $[0, \pi]$ 上有 7 个零点

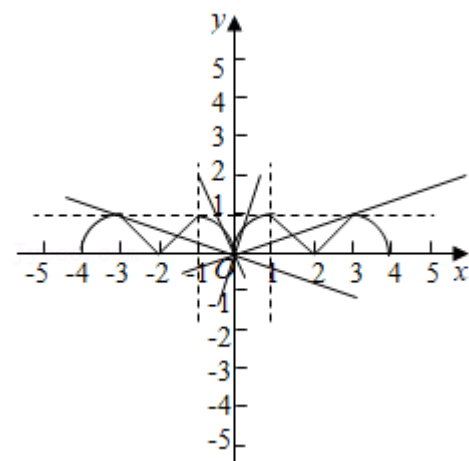
则 $y = \cos(2\omega x - \frac{\pi}{6}) \quad y = -\frac{1}{2}$ 有 7 个解

$$\frac{20\pi}{3} \leq 2\omega x - \frac{\pi}{6} < \frac{22\pi}{3}$$

$f(x) = \begin{cases} -x^2 + 2x & 0 \leq x \leq 2 \\ 2-x & 1 < x < 2 \end{cases}$

$y = f(x)$

$f(x) - (2t+1)x = 0$



$f(x) = -2x + 2$

$y = f(x)$

$-\frac{1}{3} < t < \frac{1}{2}$

B

$f(x+1) = f(-x+1)$

$f(2019) =$

A

B

C

D

$f(x+1) = f(-x+1)$

$f(x) = f(-x)$

$f(x+2) = f(-x) = f(x)$

$f(x+4) = f(x+2) = f(x)$

函数 $f(x)$ 的图像关于 $x=4$ 对称

且 $f(x)$ 在 R 上为奇函数

$$\therefore f(0)=0$$

$$\forall x \in [0, 1] \quad f(x) = a^x - 1$$

$$\therefore f(0) = a^0 - 1 = 0 \quad a = 1$$

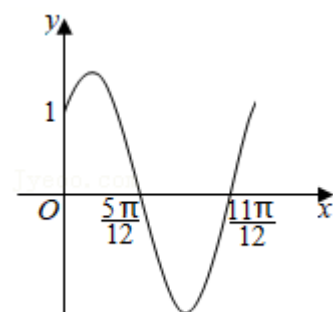
$$\therefore \forall x \in [0, 1] \quad f(x) = e^x - 1$$

$$\therefore f(2019) = (4 \times 505 - 1) = f(-1) = -(e^{-1} - 1) = 1 - e^{-1}$$

函数 C

函数 $f(x) = A \sin(\omega x + \varphi)$ ($x \in R, \omega > 0, 0 < \varphi < \frac{\pi}{2}$) 的图像如图所示

2021 • 已知函数 $f(x) = A \sin(\omega x + \varphi)$ ($x \in R, \omega > 0, 0 < \varphi < \frac{\pi}{2}$) 的图像如图所示，求 $f(x)$ 的解析式



A 2π

B $\omega = 2$

C $\varphi = \frac{\pi}{3}$

D $A = \frac{3}{2}$

$$T = 2 \left(\frac{11\pi}{12} - \frac{5\pi}{12} \right) = \pi \quad \omega = \frac{2\pi}{T} = 2$$

$$\left(\frac{5\pi}{12}, 0 \right) \quad A \sin \left(2 \times \frac{5\pi}{12} + \varphi \right) = 0 \quad \sin \left(\frac{5\pi}{6} + \varphi \right) = 0$$

$$0 < \varphi < \frac{\pi}{2} \quad \frac{5\pi}{6} < \frac{5\pi}{6} + \varphi < \frac{4\pi}{3} \quad \frac{5\pi}{6} + \varphi = \pi \quad \varphi = \frac{\pi}{6}$$

点 $(0,1)$ 在圆 $A \sin \frac{\pi}{6} = 1$ 上, 则 $A = 2$

在四面体 $ABCD$ 中

点 B

在四面体 $ABCD$ 中

21. 2021 • 在四面体 $ABCD$ 中, $AB \perp CD$, $AC \perp BD$, 平面 $ABC \cap$ 平面 $ADC = m$, 平面 $ABD \cap$ 平面 $ACD = n$

则 $AB \perp AC = n$, m, n 满足 ()

A $\frac{\sqrt{3}}{2}$

B $\frac{\sqrt{2}}{2}$

C $\frac{\sqrt{3}}{3}$

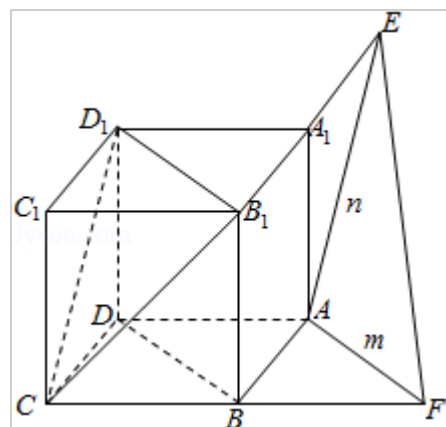
D $\frac{1}{3}$

在四面体 $ABCD$ 中, $AB \perp CD$, $AC \perp BD$, 平面 $ABC \cap$ 平面 $ADC = m$, 平面 $ABD \cap$ 平面 $ACD = n$

则 $n \parallel CD$, $m \parallel BD$, $\triangle BCD$ 为等边三角形, m, n 满足 $\angle CD \perp B = 60^\circ$

则 m, n 满足 $\frac{\sqrt{3}}{2}$

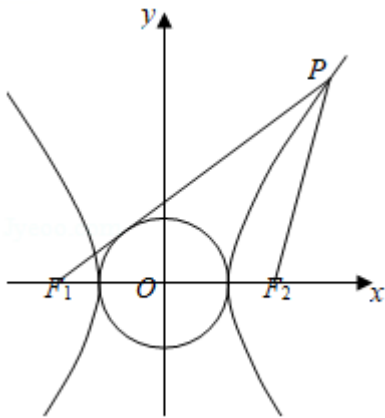
点 A



在四面体 $ABCD$ 中

22. 2021 • 在椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0)$ 中, F_1, F_2 为焦点, P 为椭圆上一点, PF_1

$x^2 + y^2 = a^2$ 则 $|PF_2| \cdot |F_1F_2|$ 的取值范围是 ()



A $\frac{5}{3}$

B $\frac{5}{4}$

C $\frac{17}{15}$

D $\frac{17}{16}$

☐ $PF_1 \perp OM$ ☐ $x^2 + y^2 = a^2$ ☐ M

☐ $|OM| = a$ ☐ $OM \perp PF_1$

☐ $PF_1 \perp NF_2$ ☐ NF_2

☐ $|PF_2| = |F_1F_2| = 2c$ ☐ $NF_2 \perp PF_1$ ☐ $|NP| = |NF_1|$

☐ $|NF_2| = 2|OM| = 2a$

☐ $|NP| = \sqrt{4c^2 - 4a^2} = 2b$

☐ $|PF_1| = 4b$

☐ $|PF_1| \cdot |PF_2| = 2a$

☐ $4b \cdot 2c = 2a$ ☐ $2b = c + a$

$4b^2 = (c + a)^2$ ☐ $4(c^2 - a^2) = (c + a)^2$ ☐

$4(c - a) = c + a$ ☐ $3c = 5a$ ☐

☐ $e = \frac{c}{a} = \frac{5}{3}$ ☐

☐ A ☐



□□□□□

23□□2

$$[-2 \ 2]$$
$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$
 $[0 \ 4]$

[1 3]

5/5

$\square \quad \square 1 \square$

$$\therefore I \square 1$$
$$\lambda = 1, \lambda$$

$\square \square \square x \in$

111

11/11

24□□2

1111

$$(-\infty, -1) \cup (0, 1)$$
$$(-1, 0) \cup (1, +\infty)$$
$$(-1 \ 0) \cup (0 \ 1)$$
$$(-\infty, -1) \cup (1, +\infty)$$

$$g(x) = \frac{f(x)}{x} \quad g'(x) = \frac{x f'(x) - f(x)}{x^2}$$

$$\parallel \quad x > 0 \quad x f'(x) - f(x) > 0$$

$$\therefore \quad x > 0 \quad g'(x) > 0$$

$$\therefore \quad g(x) = \frac{f(x)}{x} \quad (0, +\infty)$$

$$\parallel \quad f(x)$$

$$\therefore g(-x) = \frac{f(-x)}{-x} = \frac{-f(x)}{-x} = \frac{f(x)}{x} = g(x)$$

$$\therefore \quad g(x)$$

$$g(x) \quad (-\infty, 0)$$

$$f(-1) = 0 \quad g(-1) = 0$$

$$\parallel \quad f(x) > 0 \Leftrightarrow x g(x) > 0$$

$$\therefore \begin{cases} x > 0 \\ g(x) > 0 \end{cases} \begin{cases} x < 0 \\ g(x) < 0 \end{cases} \begin{cases} x > 0 \\ g(x) > g(1) \end{cases} \begin{cases} x < 0 \\ g(x) < g(-1) \end{cases}$$

$$x > 1 \quad -1 < x < 0$$

$$\therefore \quad f(x) > 0 \quad x \in (-1, 0) \cup (1, +\infty)$$

$$B$$

$$25 \text{ } 2021 \text{ } \bullet \text{ } X \cap Y \text{ } X \setminus Y = \{x \mid x \in X \text{ } x \notin Y\} \text{ } |X| \text{ } X \text{ } x = \{1$$

$$2 \times 3 \times 4 \times Y = \{3 \times 4 \times 5\} \quad |(X \setminus Y) \cup (Y \setminus X)| = (\quad)$$

$$A \times 3$$

$$B \times 4$$

$$C \times 5$$

$$D \times 6$$

$$\text{ } \quad X = \{1 \times 2 \times 3 \times 4\} \quad Y = \{3 \times 4 \times 5\}$$

$$D_{f(x)}(-\pi, \pi) \text{ 有 } 2 \text{ 个极值点}$$

证明：设 $R = \{x \mid f(x) = 0\}$

$$A: f(-x) + f(x) = (e^{-x} - e^x) \cos(-x) + (e^x + e^{-x}) \cos x = 0 \quad f(x) \text{ 在 } A \text{ 上恒为 } 0$$

$$B: f(0) = 0, \quad f(2\pi) = (e^{2\pi} - e^{-2\pi}) \cos 2\pi = e^{2\pi} - e^{-2\pi} \neq 0 = f(0) \quad B \text{ 不成立}$$

$$C: f(x) = e^x(\cos x - \sin x) + e^{-x}(\cos x + \sin x) \quad f\left(\frac{\pi}{6}\right) = e^{\frac{\pi}{6}}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) + e^{-\frac{\pi}{6}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) > 0 \quad f(x) \text{ 在 } (0, \pi) \text{ 上恒大于 } 0$$

$$\frac{\pi}{6} \text{ 是 } C \text{ 的一个极值点}$$

$$D: f(x) = (e^x + e^{-x}) \cos x - (e^x - e^{-x}) \sin x = 0 \quad f(x) \text{ 在 } x=0 \text{ 处取得极值}$$

$$g(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \quad (x \neq 0)$$

$$h(x) = \tan x \quad g(x) = 1 - \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2 \quad e^x + e^{-x} > |e^x - e^{-x}| > 0 \quad \left|\frac{e^x + e^{-x}}{e^x - e^{-x}}\right| > 1 \quad g(x) < 0 \quad g(x) \text{ 在 } (-\pi, \pi) \text{ 上恒小于 } 0$$

$$(-\infty, 0) \cup (0, +\infty) \quad x > 0 \quad x \rightarrow 0^+ \quad e^x - e^{-x} \rightarrow 0^+ \quad \frac{e^x + e^{-x}}{e^x - e^{-x}} \rightarrow +\infty \quad x \rightarrow +\infty \quad e^x \rightarrow +\infty$$

$$e^x \rightarrow 0 \quad \frac{e^x + e^{-x}}{e^x - e^{-x}} \rightarrow -1 \quad g(x) \text{ 在 } (-\pi, \pi) \text{ 上恒大于 } -1 \quad y = g(x) \quad y = h(x) \quad (-\pi, \pi)$$

$$f(x) \text{ 在 } (-\pi, \pi) \text{ 上有 } 2 \text{ 个极值点}$$

$$AD$$



A $f(x)$

$f(x)$

B□□□ □□□□□□□□□□

$C_{\alpha\beta}$ $-e < k < 0$ $f(x) = k$ $\alpha\beta\gamma\delta\epsilon\zeta\eta\theta$

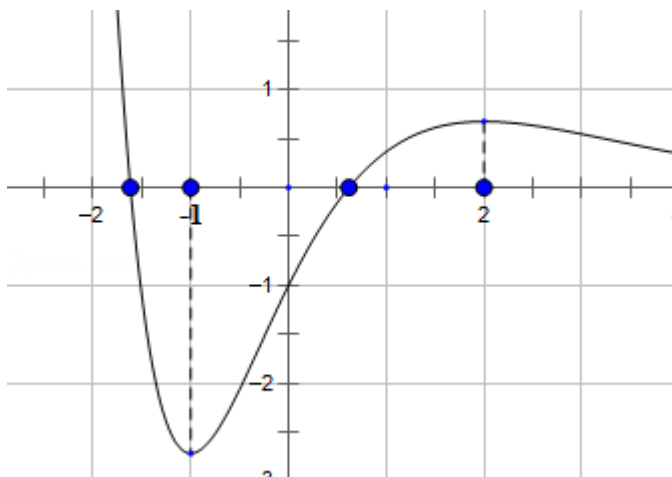
$$\sup_{X \in [t, +\infty)} f(X)_{\max} = \frac{5}{e^2} t \quad (2)$$

11

$$f(x) = e^{-x} f(1) = e^{-x} \cdot 5 = 5e^{-x}$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} 5e^{-x} = +\infty$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} 5e^{-x} = 0$$



△ABC 的面积为 D

△ABC 的面积为

△ABC 的面积为

31. 2021 • 已知函数 $f(x) = \sin(\omega x + \phi)$ 的图象如图所示，其中 $\omega > 0$ ， $0 < \phi < \pi$ 。若 $f(x)$ 的图象与直线 $y = 1$ 的交点中，相邻两交点之间的距离为 π ，则 ω 的值为 $\frac{1}{2}$ 。

△ABC 的面积为



A. 4

B. $h = -60 \cos\left(\frac{\pi}{15}t\right) + 68$

C. t_1, t_2 的取值范围是 $[0, 30]$

D. $t_1, t_2 \in [0, 20]$ 的取值范围是 $[0, 90]$

△ABC 的面积为 128 ，则 120 的值为

△ABC 的面积为 $128 - 120 = 8$ ，则 A 的值为

$$A \square\square\square\square \quad MENF \perp \square\square \quad BDD \perp B_1$$

$$B \square\square\square\square \quad MENF \square\square\square\square\square\square\square\square 1$$

$$C \square\square\square\square \quad MENF \square\square\square\square\square\square\square\square [4 \quad 4\sqrt{2}]$$

$$D \square\square\square\square \quad C_1 - MENF \square\square\square\square\square\square$$

$$\square\square\square\square\square\square\square\square\square A \square\square\square EF \square AC \square BD \square B_1 D_1 \square\square\square\square\square\square\square$$

$$\square\square\square\square\square\square\square\square\square AC \perp \square\square \quad BDD \perp B_1 \square$$

$$\square\square EF \square F \square\square\square\square A A_1 \square C C_1 \square\square\square\square$$

$$\therefore EF \parallel AC \square\square \therefore EF \perp \square\square \quad BDD \perp B_1 \square$$

$$\square\square EF \subset \square\square \quad MENF \square$$

$$\therefore \square\square \quad MENF \perp \square\square \quad BDD \perp B_1 \square\square\square\square A \square\square\square$$

$$\square\square\square\square B \square\square\square\square A \square\square\square EF \perp MN \square$$

$$\therefore \square\square\square \quad MENF \square\square\square\square \frac{1}{2} |MN| |EF| = \frac{\sqrt{2}}{2} |MN| \square$$

$$\square\square M \square N \square\square\square\square BB_1 \square DD_1 \square\square\square\square |MN| \square\square\square\square \sqrt{2} \square$$

$$\therefore \square\square\square \quad MENF \square\square\square\square\square\square\square\square 1 \square\square\square\square B \square\square\square$$

$$\square\square\square\square C \square\square\square\square\square\square\square\square\square\square EM \parallel NF \square EN \parallel MF \square$$

$$\therefore \square\square\square \quad MENF \square\square\square\square$$

$$\therefore \square\square\square \quad MENF \square\square \quad L(x) = 4 |EM| = 4 \sqrt{1^2 + (\frac{1}{2} - x)^2} \square$$

$$\square\square \quad x \in [0 \quad 1] \square\square \therefore L(x) \in [4 \quad 2\sqrt{5}] \square\square\square\square C \square\square\square$$

$$\square\square\square\square D: V_{C_1 - MENF} = V_{C_1 - MEIN} + V_{C_1 - MFN} = 2 V_{C_1 - MFN}$$

$$= 2 V_{N - C_1 MF} = 2 \times \frac{1}{3} \times S_{\triangle C_1 MF} \times D_1 C_1$$

② $a_1 + a_2 + \dots + a_{s_0} = 9$ ☐

③ $101, (a_1 + 1)^2 + (a_2 + 1)^2 + \dots + (a_{s_0} + 1)^2 \text{ „ } 111$ ☐

□□□□□□□□□□ $\{a_n\}$ □ $a_1^2 + a_2^2 + \dots + a_{s_0}^2$ □□ k □□□□□□□□ $k = \underline{\quad 6 \quad}$ □

□□□□□□□ a_1 □ a_2 □ \dots □ a_{s_0} □□ s □□□□ 0 □

□□□□2□□□□□ 1 □□□□ $\frac{50 - s - 9}{2} + 9$ □□□□ - 1 □□□□ $\frac{50 - s - 9}{2}$ □

□□□□□3□□ $101, s + 4(\frac{50 - s - 9}{2} + 9), 111$ □

□□ 7, s , 17 □

□□□ s □□□□□□

□ $s = 7$ □9□11□13□15□17□□□□□ 6 □□□□ $a_1^2 + a_2^2 + \dots + a_{s_0}^2 = 50 - s$ □

□□□□□6□

□□□□□□□□□□□□□□□□□□

35□□2021 □•□□□□□□□□□□□ $f(x) = x^2 - 5x$ □□□□□□□□

① $f(x)$ □□□□□□□□□□

② $f(x)$ □ $(\sqrt{2} \text{ „ } +\infty)$ □□□□□□

③ □□ $y = |f(x)|$ □□ 6 □□□□□□

④ □□ $|f(x)| = 3\sqrt{2}$ □□ 6 □□□□□

□□□□□□□□□□ □①②④□

□□□□□□□□①□ $f(x)$ □□□□□ R □

□ $f(-x) = -x^2 + 5x = -f(x)$ □□ $f(x)$ □□□□□□

$\therefore f(x)$ □□□□□□□□□□①□□□□

② $f(x) = 3x^2 - 5$ $x > \sqrt{2}$ $f(x) > 1 > 0$

$\therefore f(x)$ $(\sqrt{2}, +\infty)$ ②

③ $f(x) = 0$ $x = \pm\sqrt{\frac{5}{3}}$

$f(x)$ $(-\infty, -\sqrt{\frac{5}{3}})$ $(\sqrt{\frac{5}{3}}, +\infty)$ $(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}})$

$f(x) = 0$ $x = 0$ $x = \pm\sqrt{5}$

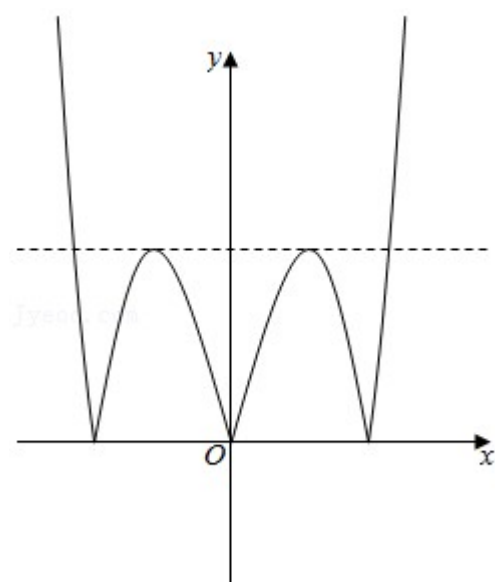
$y = |f(x)|$ $y = f(x)$ 5 ③

④ $y = f(x)$ $y = |f(x)|$

$\therefore y = |f(x)|$ $|f(\sqrt{\frac{5}{3}})| = \frac{10\sqrt{15}}{9} > 3\sqrt{2}$

$\therefore |f(x)| = 3\sqrt{2}$ 6 ④

①②④



①②④

36 2021 • $\triangle ABC$ A B C a b c $3a^2 = (c+b)(c-b)$ $\tan A \cdot \tan B$

$$\text{---} \quad (0, \frac{1}{2}) \quad \text{---}$$

$$3a^2 = c^2 - b^2$$

$$3a^2 + b^2 = c^2 = a^2 + b^2 - 2ab\cos C$$

$$a = -b\cos C$$

$$\sin A = -\sin B\cos C$$

$$\sin C\cos B + \cos C\sin B = -\sin B\cos C$$

$$\sin C\cos B = -2\sin B\cos C$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\tan A = \frac{\tan B}{1 + 2\tan^2 B}$$

$$\tan A \tan B = \frac{\tan^2 B}{1 + 2\tan^2 B} = \frac{1}{2 + \frac{1}{\tan^2 B}} \in (0, \frac{1}{2})$$

37 2021 • (1601 - 1665) 1643 “

” $\triangle ABC$ 120°

$\angle APB = \angle BPC = \angle CPA = 120^\circ$ P $P \in \triangle ABC$ $AC \perp BC$ $|PA| + |PB| = \lambda |PC|$

$$\lambda \text{---} 2 + 2\sqrt{3} \text{---}$$

$$|PA| = m|PC| \quad |PB| = n|PC| \quad |PC| = x \quad m > 0 \quad n > 0 \quad x > 0$$

$$|AC|^2 = x^2 + m^2 x^2 - 2mx^2 \cos 120^\circ = (m^2 + m + 1)x^2$$

$$|BC|^2 = x^2 + n^2 x^2 - 2nx^2 \cos 120^\circ = (n^2 + n + 1)x^2$$

$$|AB|^2 = m^2 x^2 + n^2 x^2 - 2mnx^2 \cos 120^\circ$$

$$|AB|^2 = |CA|^2 + |CB|^2$$



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$$\log(x+2y) = \log x + \log(2y)$$

$$\frac{xy + x + 2y^2}{y} = x + 2y + \frac{x}{y} = (x + 2y)\left(\frac{1}{x} + \frac{1}{2y}\right) + \frac{x}{y} = \frac{3x}{2y} + \frac{2y}{x} + 3$$

$$\begin{cases} \frac{3x}{2y} = \frac{2y}{x} \\ x + 2y = 2xy \end{cases}$$

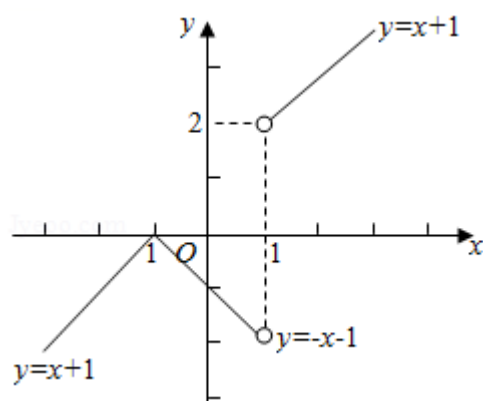
□□□□□ $2 + 2\sqrt{3}$ □

40 2021 • $y = \frac{|x^2 - 1|}{x^2 - 1}$ $y = kx$ k $(0, 1) \cup (1, 2)$

$$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{} \qquad \boxed{X+1, \quad X < -1} \qquad \boxed{}$$

☐ ☐ ☐ ☐ ☐ ☐ $y=kx$ ☐ ☐ ☐ k ☐ ☐ $0 < k < 1$ ☐ $1 < k < 2$ ☐ ☐

$$\square\square\square\square (0 \square 1) \cup (1 \square 2) \square$$



41 2021 • D 1 $f(x)$ D

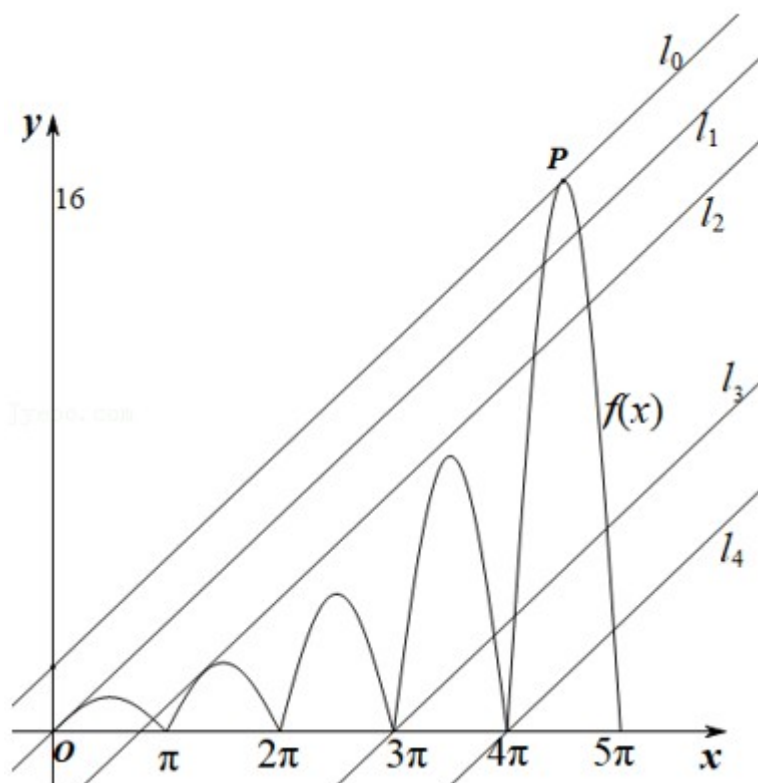
$$2\pi \int_0^{\frac{\pi}{6}} f(x) dx = \frac{\pi}{6} \int_0^{\frac{\pi}{6}} f(x) dx$$

① $\sqrt{3}$

② $\frac{\sqrt{3}}{2}$

③ $\frac{\sqrt{3}}{3}$

④0



① 函数 $f(x)$ 在 $[0, 5\pi]$ 上的最大值 $f(\frac{9\pi}{2}, 16)$ 点 P 是 1 条直线 l_0 的切点 $y = (0, 16 - \frac{9\pi}{2})$ $16 - \frac{9\pi}{2} > 0$

所以 l_0 与 $f(x)$ 有 1 个公共点

② 直线 l_0 与函数 $f(x)$ 有 1 个公共点 l_1 与函数 $f(x)$ 有 2 个公共点 $x \in [0, \frac{\pi}{2}]$ $f(x) = \cos x$

$$f'(x) = -\sin x = 1 \quad x = 0 \quad (0, 0)$$

直线 l_1 $y = x$ $f(\frac{9\pi}{2}) = 16 > \frac{9\pi}{2}$ 直线 l_1 与 $f(x)$ 有 2 个公共点 $(0, 4\pi)$ 直线 l_1 与 $f(x)$ 有 3 个公共点

所以 3 个公共点

③ 直线 l_1 与函数 $f(x)$ 有 3 个公共点 $f(x)$ 在 $[0, 5\pi]$ 上有 3 个公共点

直线 l_2 与函数 $f(x)$ 有 4 个公共点 (x_2, y_2) $x_2 \in (\pi, 2\pi)$ $f(x) = -2\cos x$ $f(x_2) = -2\cos x_2 = 1$

$$\cos x_2 = -\frac{1}{2} \quad x_2 = \frac{4\pi}{3} \quad f(\frac{4\pi}{3}) = -2\sin\frac{4\pi}{3} = \sqrt{3} \quad (\frac{4\pi}{3}, \sqrt{3}) \quad l_2: y = x - \frac{4\pi}{3} + \sqrt{3}$$

$$f(x) \in [0, 5\pi] \quad m \in [0, \frac{4}{3}\pi - \sqrt{3})$$

$$④ \quad I_2 \text{ (3}\pi, 0) \quad I_3 \quad y = x - 3\pi \quad f(x) \text{ 3 } I_3$$

$$⑤ \quad I_3 \text{ (4}\pi, 0) \quad I_4 \quad y = x - 4\pi \quad f(x) \text{ 3 } I_3 \text{ (} I_4 \text{)}$$

$$m \in (3\pi, 4\pi)$$

$$⑥ \quad I_4$$

$$m \in [0, \frac{4}{3}\pi - \sqrt{3}) \cup (3\pi, 4\pi)$$

$$m > 0 \quad x \in [1, +\infty) \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0 \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0$$

$$43 \text{ } 2021 \text{ } \bullet \quad m > 0 \quad x \in [1, +\infty) \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0 \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0$$

$$m > 0 \quad x \in [1, +\infty) \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0 \quad m \cdot 2^{m+1} - \log_{\sqrt{e}} x, 0$$

$$2^{m+1} - \frac{1}{m} \log_2 x, 0 \quad (2^m)^x - \log_{2^m} x, 0 \quad a = 2^m \quad a > 1 \quad x \in [1, +\infty) \quad a^x - \log_a x, 0 \quad a^x, \log_a x$$

$$y = a^x \quad y = \log_a x$$

$$m \quad y = a^x \quad y = \log_a x$$

$$y = a^x \quad y = \log_a x \quad y = x$$

$$m \quad y = a^x \quad y = \log_a x \quad y = x$$

$$y = \log_a x \quad y = \frac{1}{x \ln a} \quad (t \log_a t) \quad \frac{1}{t \ln a} = 1 \quad t = \frac{1}{\ln a} \text{ ①}$$

$$y = a^x \quad y = a^x \ln a \quad (t, a) \quad a^x \ln a = 1$$

$$\begin{cases} a = \log_a t \\ a \ln a = \frac{1}{t \ln a} = 1 \end{cases} \quad t = e$$

③ 已知 a, b, c 满足 $2b = a + c$

求证 $4 \leq \frac{a}{b} + \frac{b}{a} + \frac{b}{c} + \frac{c}{b} \leq 5$

$2 \times 5 = 1 + 9 = 2 + 8 = 3 + 7 = 4 + 6$ 求证

$1 \times 5 \times 9 \times 2 \times 5 \times 8 \times 3 \times 5 \times 7 \times 4 \times 5 \times 6$ 能被 3 整除

④ 已知 $1 \times 2 \times 4 \times \dots \times 1 \times 3 \times 9 \times \dots$

求证 $3 \times 5 \times 7 \times \dots \times 2 \times 5 \times 8 \times \dots$ 能被 4 整除

已知①②③

1	2	4
3	5	7
9	8	6

1	2	4
3	6	5
9	7	8

1	2	3
4	5	6
7	8	9

已知函数 $f(x)$ 在 (a, b) 上二阶可导，且 $f'(a) = f'(b) = 0$ ，求证

45 2021 年 • 已知函数 $f(x)$ 在 (a, b) 上二阶可导，且 $f'(a) = f'(b) = 0$ ，求证 $f''(x) < 0$

已知 $f(x)$ 在 (a, b) 上二阶可导

$f\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) \leq \frac{f(x_1) + f(x_2) + \dots + f(x_n)}{n}$ 其中 $n \in \mathbb{N}$, $x_1, x_2, \dots, x_n \in (a, b)$ 且 $f(x) = \sin x$, $f'(x) =$

$-\sin x$ 求证 $x_1 + x_2 + x_3 = \pi$ 时 $\sin x_1 + \sin x_2 + \sin x_3$ 的最大值为

已知 $f(x) = \sin x$, $x \in (0, \pi)$, $f'(x) = \cos x$, $f''(x) = -\sin x$

求证 $f\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) \leq \frac{f(x_1) + f(x_2) + \dots + f(x_n)}{n}$

$$\sin x_1 + \sin x_2 + \sin x_3, 3\sin\left(\frac{x_1 + x_2 + x_3}{3}\right) = 3 \times \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2}$$

$$\therefore \sin A + \sin B + \sin C \leq \frac{3\sqrt{3}}{2}$$

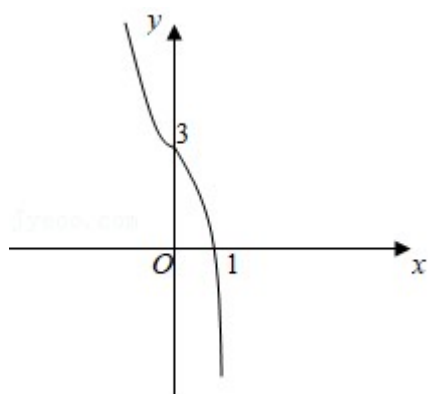
$$\sin x \leq \frac{3\sqrt{3}}{2}$$

46. 2021 年 1 月 1 日起，全国范围内全面实施机动车限行措施，

$$f(x) = \begin{cases} x^2 - 4x + 3, & x \leq 0 \\ -x^2 - 2x + 3, & x > 0 \end{cases} \quad f(x+a) > f(2a-x) \quad [a, a+1] \quad a$$

$$(-\infty, -2)$$

$$f(x) = \begin{cases} x^2 - 4x + 3, & x \leq 0 \\ -x^2 - 2x + 3, & x > 0 \end{cases}$$



$$f(x+a) > f(2a-x) \quad [a, a+1]$$

$$x+a < 2a-x \quad x \in [a, a+1]$$

$$a > 2x \quad x \in [a, a+1]$$

$$\therefore a > 2(a+1) \quad a < -2$$

$$(-\infty, -2)$$

48 2021 • $\{a_n\}$ $d \neq 0$ $a_1 = 1$ $a_1, a_2, a_3, \dots, a_k, a_{k+1}, a_{k+2}, \dots$

$$a_{k_n} \dots n \in N \quad \frac{a_n}{2k_n - 1} \cdot \frac{a_m}{2k_m - 1} (m \in N) \quad m = \underline{1} \quad \underline{2}$$

$$\{a_n\} \quad a_1 = 1 \quad a_1, a_2, a_3, \dots$$

$$\therefore (1+d)^2 = 1 \times (1+4d) \quad d \neq 0 \quad d = 2$$

$$\therefore a_n = 1 + 2(n-1) = 2n-1$$

$$a_1, a_2, a_{k-1}, a_{k-2}, \dots, a_{k-n}, \dots$$

$$1 \dots 3$$

$$\therefore a_{k_n} = 3^{n+1}$$

$$a_n = 2n-1 \quad a_{k_n} = 2k_n-1$$

$$\therefore 2k_n-1 = 3^{n+1}$$

$$\therefore k_n = \frac{1}{2}(3^{n+1} + 1)$$

$$n \in N \quad \frac{a_n}{2k_n-1} \cdot \frac{a_m}{2k_m-1} (m \in N)$$

$$\frac{2n-1}{3^{n+1}} \cdot \frac{2m-1}{3^{m+1}}$$

$$f(n) = \frac{2n-1}{3^{n+1}} > 0 \quad \frac{f(n+1)}{f(n)} = \frac{\frac{2n+1}{3^{n+2}}}{\frac{2n-1}{3^{n+1}}} = \frac{1}{3} \cdot \frac{2n+1}{2n-1} > 1$$

$$\therefore n=1, n=2, \dots, f(n) \dots n, 2, \dots, f(n) \dots$$

$$n \in N \quad \frac{a_n}{2k_n-1} \cdot \frac{a_m}{2k_m-1} (m \in N) \quad m=1, 2$$

④ $y = \cos^2 x - \sin^2 x + \sin(\frac{\pi}{2} - x)$

$\alpha \in (0, \frac{\pi}{2})$ $\sin \alpha + \cos \alpha > 1$ ①

② $y = \tan x$ $(k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2}), k \in \mathbb{Z}$ ②

③ $f(x) = \sin(x + \frac{\pi}{6})$ 2π $g(x) = |\sin(x + \frac{\pi}{6})|$

$g(x + \pi) = |\sin(x + \frac{\pi}{6} + \pi)| = |\sin(x + \frac{\pi}{6})| = g(x)$ ③

④ $f(x) = 2\cos^2 x + \cos x - 1$ $f(-x) = f(x)$ $t = \cos x \in [-1, 1]$

$y = 2t^2 + t - 1 = 2(t + \frac{1}{4})^2 - \frac{9}{8}$

$t = -\frac{1}{4}$ $y_{\min} = -\frac{9}{8}$ $t = 1$ $y_{\max} = 2$ ④

③ ④

51 2021 • $f(x)$ R $x \in R$ $f(x+1) = -f(x)$ $x \in [0, 1]$

$f(x) = 3^x$

① 2 $f(x)$

② $f(x)$ (2,3)

③ $f(x)$ 1 0

④ $x = 2$ $f(x)$

① ② ④

$x \in R$ $f(x+1) = -f(x)$ $f(x+2) = f(x)$ 2 ①

$x \in (2,3)$ $x - 2 \in (0,1)$ $x \in [0,1]$ $f(x) = 3^x$ 2 $f(x)$ (2,3) ②

$$\begin{cases} -2x_1 + z_1 = 0 \\ 2x_1 = 0 \end{cases} \begin{cases} -2x_2 + z_2 = 0 \\ -2x_2 + by_2 = 0 \end{cases}$$

$$x_1 = 0, n_1 = (0, 1, 0), z_2 = 2, n_2 = (1, \frac{2}{b}, 2)$$

$$\therefore n_1 \cdot n_2 = \frac{2}{b}, |n_1| = 1, |n_2| = \sqrt{5 + \frac{4}{b^2}}$$

$$\parallel \text{ } Q \text{ } PD \text{ } A \text{ } \frac{\pi}{4}$$

$$\therefore \cos \langle n_1, n_2 \rangle = \frac{n_1 \cdot n_2}{|n_1| |n_2|} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{5 + \frac{4}{b^2}}} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{5 + \frac{4}{b^2}}} = \frac{\sqrt{2}}{2}, b = \frac{2\sqrt{5}}{5}$$

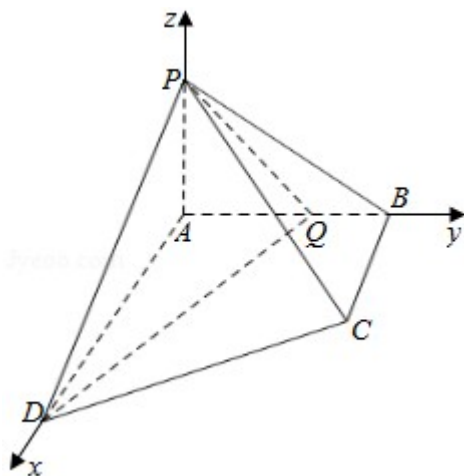
$$\therefore S_{\triangle ADQ} = \frac{1}{2} AD \cdot AQ = \frac{1}{2} \times 2 \times \frac{2\sqrt{5}}{5} = \frac{2\sqrt{5}}{5}$$

$$S_{\square ABECD} - S_{\triangle ADQ} = \frac{1}{2} \times (1+2) \times 1 - \frac{2\sqrt{5}}{5} = \frac{3}{2} - \frac{2\sqrt{5}}{5}$$

$$\square S_1 < S_2 \therefore S_1 = \frac{3}{2} - \frac{2\sqrt{5}}{5}, S_2 = \frac{2\sqrt{5}}{5}$$

$$\therefore S_1 : S_2 = (3\sqrt{5} - 4) : 4$$

$$(3\sqrt{5} - 4) : 4$$



$$\Delta f(x) = x^2 - (a-1)x + 1 \quad \frac{a-1}{2} > 0 \quad 1 - \left(\frac{a-1}{2}\right)^2 < 0 \quad a > 3$$

$$\frac{3a-2}{4} > 0$$

$$y = \left(2 - \frac{3a}{4}\right)x \quad f(x) = x^2 - (a-1)x + 1$$

$$(x_0, \left(2 - \frac{3a}{4}\right)x_0)$$

$$f(x) = 2x - (a+1)$$

$$\begin{cases} \left(2 - \frac{3a}{4}\right)x_0 = x_0^2 - (a-1)x_0 + 1 \\ 2x_0 - (a+1) = 2 - \frac{3a}{4} \end{cases} \quad x_0 = \frac{a+4}{8}$$

$$\left(2 - \frac{3a}{4}\right) \cdot \frac{a+4}{8} = \left(\frac{a+4}{8}\right)^2 - (a-1) \cdot \frac{a+4}{8} + 1$$

$$a^2 + 8a - 48 = 0 \quad a = 4 \quad a = -12$$

$$a = 4 \quad y = f(x) \quad y = g(x) \quad x > 0 \quad 3$$

$$y = f(|x|) \quad y = g(x) \quad 6$$

$$1 \quad 3 \quad 2 \quad 4$$

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